



ALMA MATER STUDIORUM  
UNIVERSITÀ DI BOLOGNA  
CAMPUS DI RIMINI

# DISEQUAZIONI DI SECONDO GRADO

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PRECORSO DI MATEMATICA GENERALE CLET E CLEI

$$ax^2 + bx + c > 0 \quad (\geq 0, \leq 0, < 0)$$

con  $a \neq 0$

Ogni disequazione di secondo grado può essere risolta graficamente, deducendo il

segno del trinomio  $ax^2 + bx + c$  dal grafico della parabola corrispondente

$$y = ax^2 + bx + c$$



## ESEMPIO

Risolvere la disequazione

$$x^2 - 5x + 6 > 0$$

significa trovare le ascisse dei punti della parabola:

$$y = x^2 - 5x + 6$$

che hanno ordinata positiva ( $y > 0$ ).



## Procedimento:

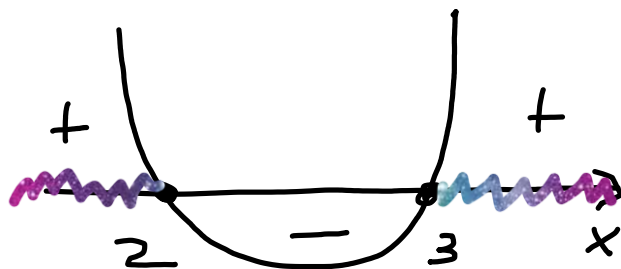
si risolve l'equazione associata:

$$x^2 - 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2}$$

↙ X=2  
↘ X=3

Si rappresenta la parabola associata:

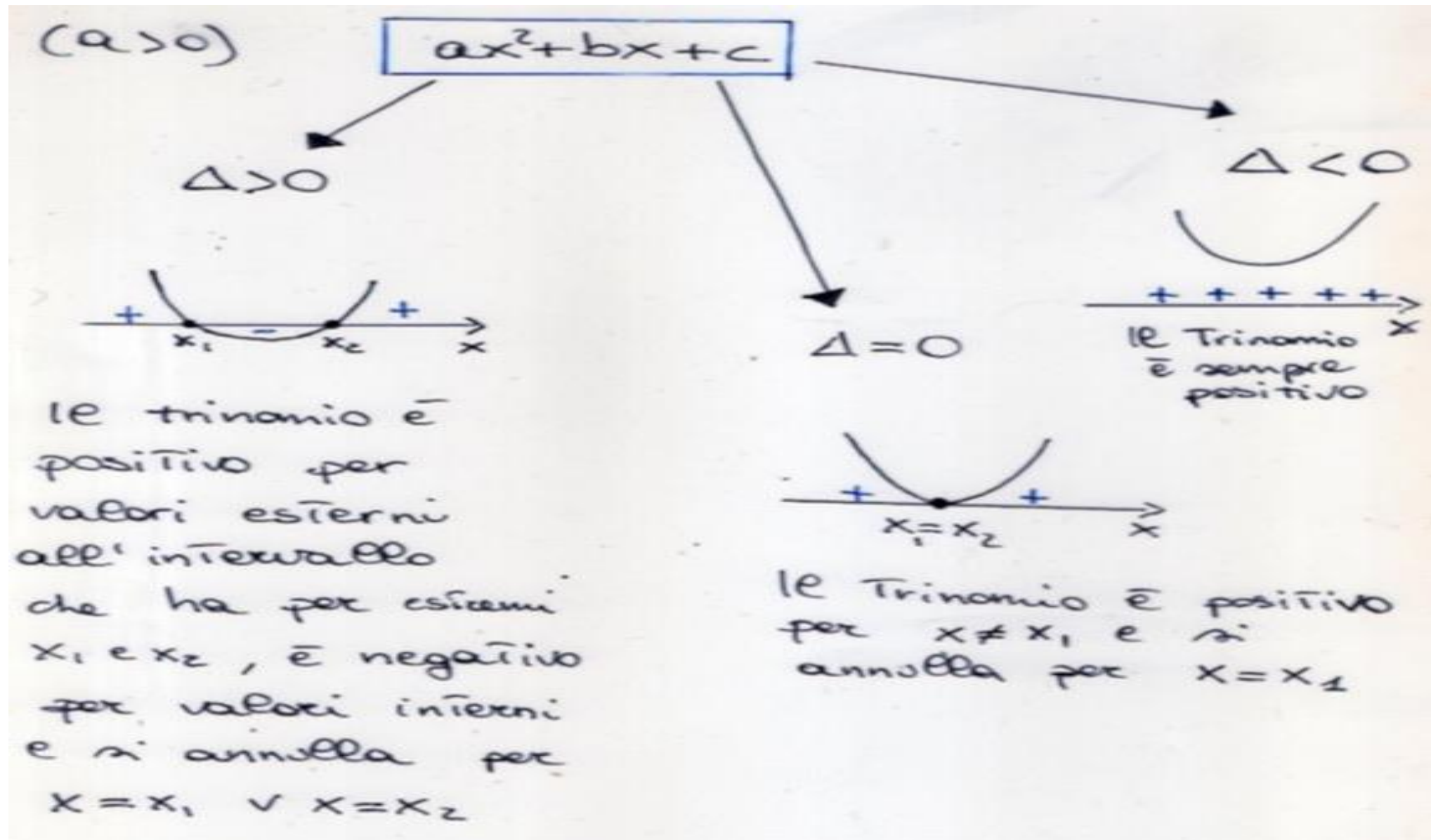


$$x < 2 \vee x > 3$$

$$S = (-\infty, 2) \cup (3, +\infty)$$



# IL SEGNO DEL TRINOMIO



## Esercizio:

$$2 + x(1-x) < 2(4x+7)$$

$$2+x-x^2 < 8x+14$$

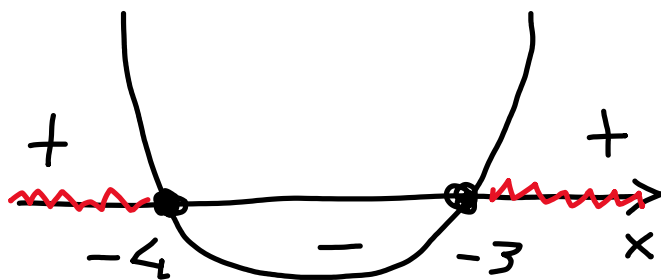
$$-x^2 - 7x - 12 < 0$$

$$x^2 + 7x + 12 > 0$$

$$x = \frac{-7 \pm \sqrt{49-48}}{2} = \frac{-7 \pm 1}{2}$$

-4

-3



$$x < -4 \quad \vee \quad x > -3$$

$$S = (-\infty, -4) \cup (-3, +\infty)$$



## ESERCIZI:

1)  $x^2 + 10x + 25 < 0$

2)  $x^2 - 4x + 5 > 0$

3)  $x^2 + x + 1 < 0$

4)  $x^2 - 8x + 16 > 0$

5)  $-4x^2 + 12x - 9 \geq 0$

4

6)  $-8x - x^2 \geq 0$

7)  $x^2 \geq 25$

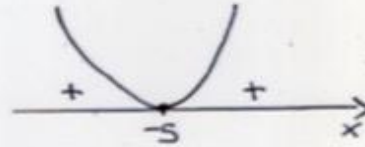


•  $x^2 + 10x + 25 < 0$

$$x^2 + 10x + 25 = 0$$

$$x = -5 \pm \sqrt{25 - 25} = -5$$

$$x_1 = x_2 = -5$$



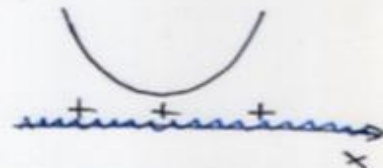
La disequazione è impossibile

$$S = \emptyset$$

•  $x^2 - 4x + 5 > 0$

$$x^2 - 4x + 5 = 0$$

$$x = 2 \pm \sqrt{4 - 5} \quad \Delta < 0$$



La disequazione è sempre verificata

$$S = \mathbb{R}$$





•  $x^2 + x + 1 < 0$

$$x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} \quad \Delta < 0$$



$$S = \emptyset$$

•  $x^2 - 8x + 16 > 0$

$$x^2 - 8x + 16 = 0$$

$$x = 4 \pm \sqrt{16-16} = 4$$

$$x_1 = x_2 = 4$$



$$\forall x \neq 4$$

$$S = \mathbb{R} \setminus \{4\} = (-\infty, 4) \cup (4, +\infty)$$



$$\bullet -4x^2 + 12x - 9 \geq 0$$

$$4x^2 - 12x + 9 \leq 0$$

$$4x^2 - 12x + 9 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{4} = \frac{6}{4} = \frac{3}{2}$$



$$\therefore x = \frac{3}{2}$$

$$S = \left\{ \frac{3}{2} \right\}$$



$$\bullet -8x - x^2 \geq 0$$

$$x^2 + 8x \leq 0$$

$$x^2 + 8x = 0$$

(Eq. associata)  
spuria

$$x(x+8) = 0$$

$$x = 0$$

✓

$$x + 8 = 0$$

⇓

$$x = -8$$



$$-8 \leq x \leq 0$$

$$S = [-8; 0]$$



$$x^2 \geq 25$$

$$x^2 - 25 \geq 0$$

$$x^2 - 25 = 0 \quad (\text{Eq. associata})$$

*puza*

$$x^2 = 25 \Rightarrow x = \pm\sqrt{25} = \pm 5$$



$$x \leq -5 \vee x \geq 5$$

$$S = (-\infty, -5] \cup [5, +\infty)$$



# DISUGUAGLIANI FRATTE

$$\frac{x-6}{x^2-16x+63} \geq 0$$

$N \geq 0$      $x-6 \geq 0 \rightarrow \boxed{x \geq 6}$

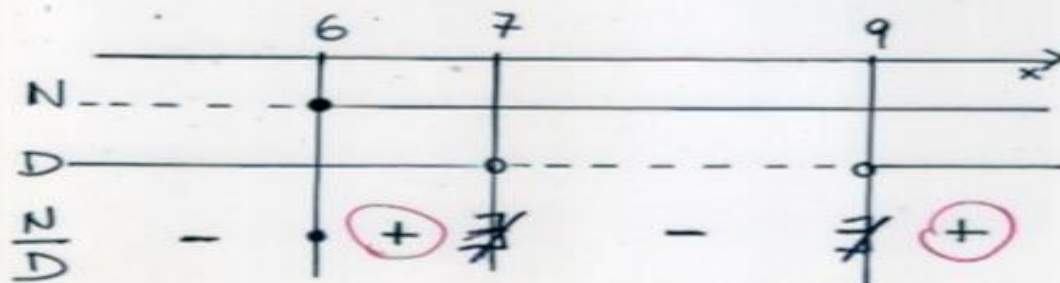
$D > 0$      $x^2-16x+63 > 0$

$$x^2-16x+63=0$$

$$x_{1,2} = 8 \pm \sqrt{64-63} = 8 \pm 1 \begin{cases} \rightarrow x_1 = 7 \\ \rightarrow x_2 = 9 \end{cases}$$



$$\boxed{x < 7 \vee x > 9}$$



$$6 \leq x < 7 \vee x > 9$$

$$S = [6, 7) \cup (9, +\infty)$$



## Esercizi:

$$1) \frac{x^2+x}{x-1} < 0$$

$$2) \frac{x^2-4}{x+2} < 2$$



$$\frac{x^2 + x}{x - 1} < 0$$

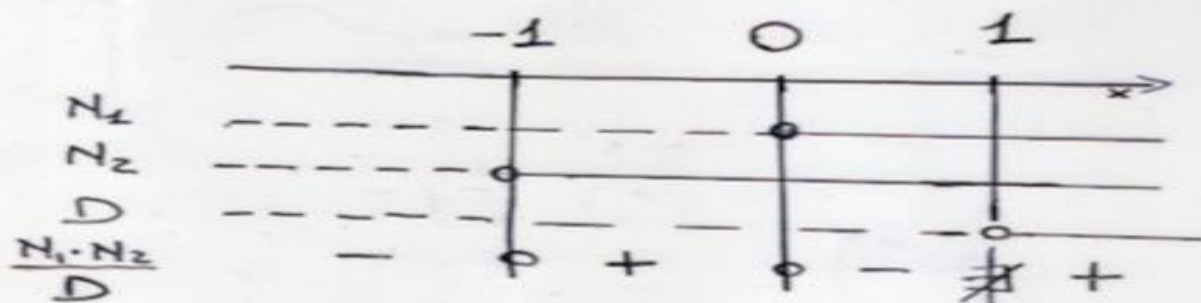
$$\frac{x(x+1)}{x-1} < 0$$

$$N > 0 \quad x(x+1) > 0$$

$$N_1 > 0 \quad \boxed{x > 0}$$

$$N_2 > 0 \quad x+1 > 0 \Rightarrow \boxed{x > -1}$$

$$D > 0 \quad x - 1 > 0 \quad \boxed{x > 1}$$



$$x < -1 \vee 0 < x < 1$$

$$S = (-\infty, -1) \cup (0, 1)$$





$$2. \quad \frac{x^2 - 4}{x + 2} < 2$$

Prava finale 28.09.12

$$\frac{x^2 - 4}{x + 2} - 2 < 0$$

$$\frac{x^2 - 4 - 2x - 4}{x + 2} < 0$$

$$\frac{x^2 - 2x - 8}{x + 2} < 0$$

$$N > 0 \quad x^2 - 2x - 8 > 0$$

$$x^2 - 2x - 8 = 0$$

$$x = 1 \pm \sqrt{1 + 8} = 1 \pm 3 \quad \begin{array}{l} -2 \\ 4 \end{array}$$



$$\boxed{x < -2 \vee x > 4}$$

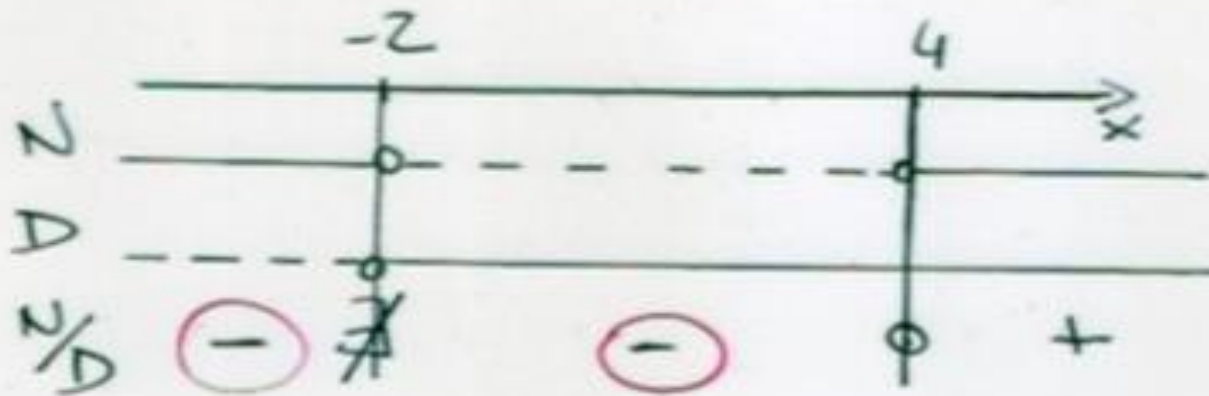
$$D > 0$$

$$x + 2 > 0$$

$$\boxed{x > -2}$$







$$x < 4 \wedge x \neq -2$$

$$S = (-\infty, -2) \cup (-2, 4)$$

